Sample Solutions

Electrical Engineering – Microsoft Word written Sample Solution

Question:

D 5.51 The NMOS transistors in the circuit of Fig. P5.51 have $V_t = 0.5$ V, $\mu_n C_{ox} = 90\ \mu A/V^2$, $\lambda = 0$, and $L_1 = L_2 = L_3 = 0.5\ \mu m$. Find the required values of gate width for each of Q_1, Q_2, and Q_3 to obtain the voltage and current values indicated.

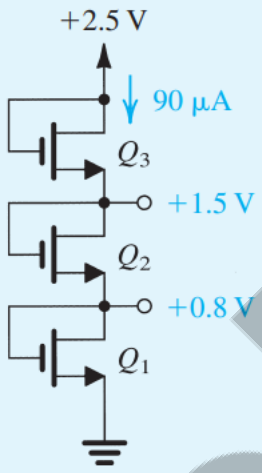


Figure P5.51

The NMOS transistors in the circuit of Fig. P5.51 have Vt = 0.5 V, µnCox = 90 µA/V^2 , λ = 0, and L1 = L2 = L3 = 0.5 µm. Find the required values of gate width for each of Q1, Q2, and Q3 to obtain the voltage and current values indicated.

Answer:

Given data and variables used:

$$Threshold\ Voltage, V_t = 0.5\ V$$

$$\mu_n C_{ox} = 90 * 10^{-6}\ \frac{A}{V^2}$$

$$Channel\ length\ modulation\ (\lambda) = 0$$

$$Body\ Effect\ (\gamma) = 0$$

$$Length, L_1 = L_2 = L_3 = 0.5\ \mu m,$$

$$Drain\ current\ of\ Q_1 =\ Q_2 = Q_3 = 90\ \mu A$$

<u>Step 1) Calculate mode of operation of MOSFET, Q_1.</u>

$$Gate\ Voltage, V_{G1} = Drain\ Voltage, V_{D1} = 0.8\ V$$

$$V_{GS1} =\ V_{G1} - V_{S1} =\ 0.8 - 0 = 1\ V$$

$$V_{GS1} - V_t =\ 0.8 - 0.5 =\ 0.3\ V$$

$$V_{DS1} = V_{D1} - V_{S1} =\ 0.8 - 0 = 0.8\ V$$

Since $V_{GS1} > V_t\ and\ V_{DS1} > V_{GS1} - V_t$, NMOS transistor, Q1 is biased in saturation region.

<u>Step 2) Apply saturation current equation to find out width, W of the MOSFET, Q1.</u>

$$I_{D1} = \frac{\mu_n C_{ox}}{2} * \frac{W_{Q1}}{L_1}(V_{GS1} - V_t)^2$$

$$90 * 10^{-6} = \frac{90 * 10^{-6}}{2} * \frac{W_{Q1}}{0.5\ \mu m}(0.3)^2$$

$$\boxed{W_{Q1} = 11.11\ \mu m}$$

<u>Step 3) Calculate mode of operation of MOSFET, Q_2.</u>

$$V_{GS2} =\ V_{G2} - V_{S2} =\ 1.5 - 0.8 = 0.7\ V$$

$$V_{GS2} - V_t =\ 0.7 - 0.5 =\ 0.2\ V$$

$$V_{DS2} = V_{D2} - V_{S2} =\ 1.5 - 0.8 = 0.7\ V$$

Since $V_{GS2} > V_t\ and\ V_{DS2} > V_{GS2} - V_t$, NMOS transistor, Q2 is biased in saturation region.

<u>Step 4) Apply saturation current equation to find out width, W of the MOSFET, Q2.</u>

$$I_{D2} = \frac{\mu_n C_{ox}}{2} * \frac{W_{Q2}}{L_2}(V_{GS2} - V_t)^2$$

$$90 * 10^{-6} = \frac{90 * 10^{-6}}{2} * \frac{W_{Q1}}{0.5\ \mu m}(0.2)^2$$

$$\boxed{W_{Q2} = 25\ \mu m}$$

Step 5) Calculate mode of operation of MOSFET, Q_3.

$$V_{GS3} = V_{G3} - V_{S3} = 2.5 - 1.5 = 1\ V$$

$$V_{GS3} - V_t = 1 - 0.5 = 0.5\ V$$

$$V_{DS3} = V_{D3} - V_{S3} = 2.5 - 1.5 = 1\ V$$

Since $V_{GS3} > V_t\ and\ V_{DS3} > V_{GS3} - V_t$, NMOS transistor, Q3 is biased in saturation region.

Step 6) Apply saturation current equation to find out width, W of the MOSFET, Q2.

$$I_{D3} = \frac{\mu_n C_{ox}}{2} * \frac{W_{Q2}}{L_2}(V_{GS2} - V_t)^2$$

$$90 * 10^{-6} = \frac{90 * 10^{-6}}{2} * \frac{W_{Q1}}{0.5\ \mu m}(0.5)^2$$

$$\boxed{W_{Q3} = 4\ \mu m}$$

Final Answers:

$$W_{Q1} = 11.11\ \mu m$$

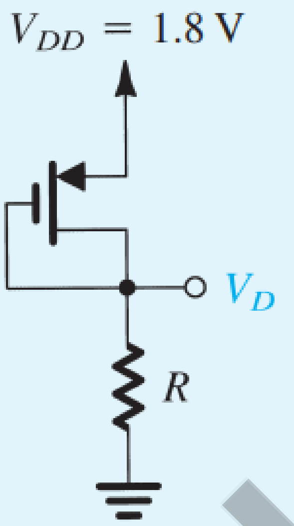
$$W_{Q2} = 25\ \mu m$$

$$W_{Q3} = 4\ \mu m$$

Electrical Engineering – Hand written Sample Solution

Question:

D 5.49 The PMOS transistor in the circuit of Fig. P5.49 has $V_t = -0.5$ V, $\mu_p C_{ox} = 100\ \mu A/V^2$, $L = 0.18\ \mu m$, and $\lambda = 0$. Find the values required for W and R in order to establish a drain current of 180 μA and a voltage V_D of 1 V.



The PMOS transistor in the circuit of Fig. P5.49 has V1 = -0.5 V, upCox = 100 uA/V^2, L = 0.18 um, and λ = 0. Find the values required for W and R in order to establish a drain current of 180 uA and a voltage VD of 1 V.

Answer:

Given data & Variables used:

Threshold Voltage, $V_t = -0.5V$, $\mu_p C_{ox} = 100*10^{-6} \frac{A}{V^2}$

Drain voltage, $V_D = 1\ V$, Drain Current, $I_D = 180\mu A$

Length, $L = 0.18\mu m$, Source voltage, $V_S = 1.8V$

channel length modulation, $\lambda = 0$, Body effect, $\gamma = 0$

step 1) Calculate mode of operation of MOSFET :→

Gate to Source Voltage, $V_{GS} = V_G - V_S = V_D - V_S = 1 - 1.8$

$V_{GS} = -0.8V$

$V_{GS} - V_t = -0.8 - (-0.5) = -0.3V$

Drain to Source Voltage, $V_{DS} = V_D - V_{DS} = V_D - 1.8 = 1 - 1.8 = -0.8V$

since $V_{GS} < V_t$ & $V_{DS} > V_{GS} - V_t$

PMOS ⟶ saturation region.

step 2) Apply saturation region current equation to Calculate width of PMOS transistor :→

$$I_D = \frac{\mu_n C_{ox}}{2} * \frac{W}{L} (V_{GS} - V_t)^2$$

$$180*10^{-6} = \frac{100*10^{-6}}{2} * \frac{W}{0.18\mu m} (-0.3)^2$$

$\boxed{W = 7.2\mu m}$

step 3) Use Drain Voltage, V_D to find out R_D.

Voltage across $R_D = I_D R_D = V_D = 1\ V$

$R_D = \frac{1}{180\mu A} \Rightarrow \boxed{R_D = 5.56 k\Omega}$

Final Answers:- $\boxed{W = 7.2\mu m}$ & $\boxed{R_D = 5.56 k\Omega}$

Math – Microsoft Word written Sample Solution

Question:

The heat conduction problem is given by:

$$u_{xx} = \alpha^2 u_t \,, \qquad 0 < x < 40 \,, t > 0$$

Find an expression for the temperature $u(x,t)$ if the initial temperature distribution in the rod is the given function and rod is insulated at the ends. Suppose that $\alpha^2 = 1$.

$$u(x,0) = \begin{cases} x & 0 \le x < 20 \\ 40 - x & 20 \le x < 40 \end{cases}$$

Answer:

Step 1) Substituting the solution $u(x,y) = X(x)T(y)$ in given differential equation.

The heat conduction problem is formulated as

$$u_{xx} = u_t \,, \qquad 0 < x < 40 \,, t > 0$$

$$Boundary\ condtions, u(0,t) = 0, \quad u(40,t) = 0, t > 0$$

$$Initial\ conditions, u(x,0) = \begin{cases} x & 0 \le x < 20 \\ 40 - x & 20 \le x < 40 \end{cases}$$

We consider solutions of the form

$$u(x,y) = X(x)T(y)$$

$$u_{xx} = X''T, \qquad u_t = XT'$$

Substituting in the given partial differential equation we get

$$X''T = XT'$$

Divide both sides of the differential equation by the product XT to obtain

$$\frac{X''}{X} = \frac{T'}{T}$$

Since both sides of the resulting equation are functions of different variables, each must be equal to a constant, say $-\lambda$

We obtain the ordinary differential equations as:

$$\frac{X''}{X} = \frac{T'}{T} = -\lambda$$

$$X'' + \lambda X = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$T' + \lambda T = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

Step 2) Solving ordinary differential equations (1) and (2).

Solving equation (1)

$$X'' + \lambda X = 0$$

Auxiliary equation is

$$m^2 + \lambda = 0$$

$$m = \pm\sqrt{\lambda} i$$

$$X = c_1 \cos(\sqrt{\lambda} x) + c_2 \sin(\sqrt{\lambda} x) \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Using boundary conditions, $u(0,t) = 0$, $u(40,t) = 0, t > 0$

$$0 = c_1 \cos(\sqrt{\lambda} 0) + c_2 \sin(\sqrt{\lambda} 0)$$

$$c_1 = 0$$

$$0 = c_1 \cos(40\sqrt{\lambda}) + c_2 \sin(40\sqrt{\lambda})$$

$$0 = c_2 \sin(40\sqrt{\lambda})$$

This implies

$$\sqrt{\lambda} = \frac{n\pi}{40} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Using value of c_1 and $\sqrt{\lambda}$ in equation (3)

The eigen-functions are given by

$$X_n = \sin\left(\frac{n\pi}{40}x\right)$$

And using equation (4) eigenvalues are given by

$$\lambda_n = \frac{n^2\pi^2}{1600}$$

Thus we can obtain the family of equations for ‘T’ by using equation (2)

$$T' + \lambda_n T = 0$$

$$\frac{dT}{dt} = -\frac{\lambda_n T}{1}$$

$$\frac{dT}{T} = -\lambda_n dt$$

Integrating both sides

$$\int \frac{dT}{T} = -\int \lambda_n dt$$

$$\ln(T) = -\lambda_n t$$

Using the value of λ_n

$$T_n = e^{-\frac{n^2\pi^2}{1600}t}$$

$$u_n(x,t) = X_n(x)T_n(t)$$

$$u_n(x,t) = e^{-\frac{n^2\eth^2}{1600}t}\sin\left(\frac{n\pi}{40}x\right) \ldots\ldots\ldots\ldots\ldots..(5)$$

<u>Step 3) Obtaining the solution of the given heat conduction problem</u>

The general solution can be given by using equation (5) as:

$$u(x,t) = \sum_{n=1}^{\infty} c_n u_n(x,t)$$

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{1}{1600}n^2\pi^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

By putting t = 0

$$u(x,0) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{40}x\right)$$

Using the initial condition

$$u(x,0) = \begin{cases} x & 0 \le x < 20 \\ 40 - x & 20 \le x < 40 \end{cases} = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{40}x\right)$$

Thus we can find Fourier coefficients c_n by using the formula:

$$c_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{40}\right)dx$$

$L = 40$

$$c_n = \frac{2}{40}\int_0^{20} x\sin\left(\frac{n\pi x}{40}\right)dx + \frac{2}{40}\int_{20}^{40}(40-x)\sin\left(\frac{n\pi x}{40}\right)dx$$

$$c_n = \frac{2}{40}\left[-x\frac{40}{n\pi}\cos\left(\frac{n\pi x}{40}\right)\right]_0^{20} + \frac{2}{40}\int_0^{20}\frac{40}{n\pi}\cos\left(\frac{n\pi x}{40}\right)dx + \frac{2}{40}\left[-(40-x)\frac{40}{n\pi}\cos\left(\frac{n\pi x}{40}\right)\right]_{20}^{40} - \frac{2}{40}\int_{20}^{40}\frac{40}{n\pi}\cos\left(\frac{n\pi x}{40}\right)dx$$

$$c_n = \frac{2}{40}\left[-\frac{800}{n\pi}\cos\left(\frac{n\pi}{2}\right) - 0\right] + \frac{2}{40}\left[\frac{1600}{n^2\pi^2}\sin\left(\frac{n\pi x}{40}\right)\right]_0^{20} + \frac{2}{40}\left[0 + \frac{800}{n\pi}\cos\left(\frac{n\pi}{2}\right)\right] - \frac{2}{40}\left[\frac{1600}{n^2\pi^2}\sin\left(\frac{n\pi x}{40}\right)\right]_{20}^{40}$$

$$c_n = -\frac{40}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2}{40}\left[\frac{1600}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) - 0\right] + \frac{40}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{40}\left[0 - \frac{1600}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$c_n = -\frac{40}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{80}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{40}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{80}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$c_n = \frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

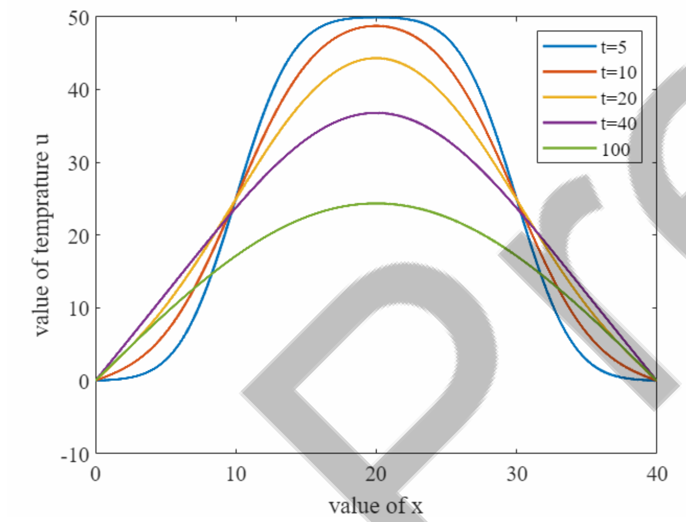
Therefore the solution of the given heat conduction problem is given by

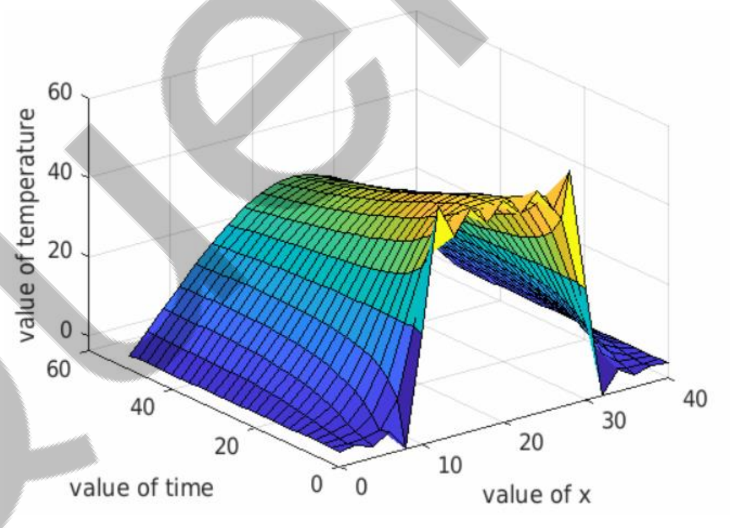
$$u(x,t) = \sum_{n=1}^{\infty} c_n u_n(x,t)$$

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{1}{1600}n^2\pi^2 t} \sin\left(\frac{n\pi}{40}x\right)$$

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} \frac{160}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) e^{-\frac{1}{1600}n^2\pi^2 t} \sin\left(\frac{n\pi}{40}x\right)}$$

MATLAB plot for $u(x,t)$





Math – Hand written Sample Solution

Question:

For the given matrix A find a basis for Col(A) and basis for Nul(A).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 6 \\ 3 & -1 & 3 \end{bmatrix}$$

Answer:

Step 1) Reduce the given matrix to echelon form using row operations

$R_2 \rightarrow R_2 - 4R_1$
$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -6 \\ 0 & -7 & -6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow (-\frac{1}{7}) * R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{6}{7} \\ 0 & 0 & 0 \end{bmatrix}$$

Step 2) Find out the basis for Col(A).

By looking at the echelon form of A, observe that there are two non-zero rows. This implies:

⇒ Maximum number of independent Columns are 2.

By looking at echelon form, we can say that Either Column 1 & Colum 2 are independent or Column 1 & Column 3 are independent.

For Col(A), two answers are possible:-

a)

Basis of Col(A) = $\left\{ \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$

b)

$\left\{ \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} \right\}$

Step 3) Find out the basis of Nul(A)

To find out the basis of Nul(A), we need to solve:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 6/7 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

we have,

$$x_1 + 2x_2 + 3x_3 = 0 \longrightarrow (1)$$

$$x_2 + \frac{6}{7}x_3 = 0 \longrightarrow (2)$$

we have 2 equations & 3 variable, so out of 3 variables, one variable will be free.

let x_3 is free variable (Any variable can be Assumed free)

then from eqn (2) $\Rightarrow x_2 = -\frac{6}{7}x_3$

Using the value of x_2 in eqn (1)

$$x_1 - \frac{12}{7}x_3 + 3x_3 = 0$$

$$x_1 = \frac{-9}{7}x_3$$

Thus

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{9}{7} \\ -\frac{6}{7} \\ 1 \end{bmatrix} x_3 = \begin{bmatrix} -9 \\ -6 \\ -7 \end{bmatrix} \frac{x_3}{7}$$

Basis of Nul(A) $\Rightarrow \left\{ \begin{bmatrix} -9 \\ -6 \\ 7 \end{bmatrix} \right\}$ (* Answer for Nul(A) is not unique).